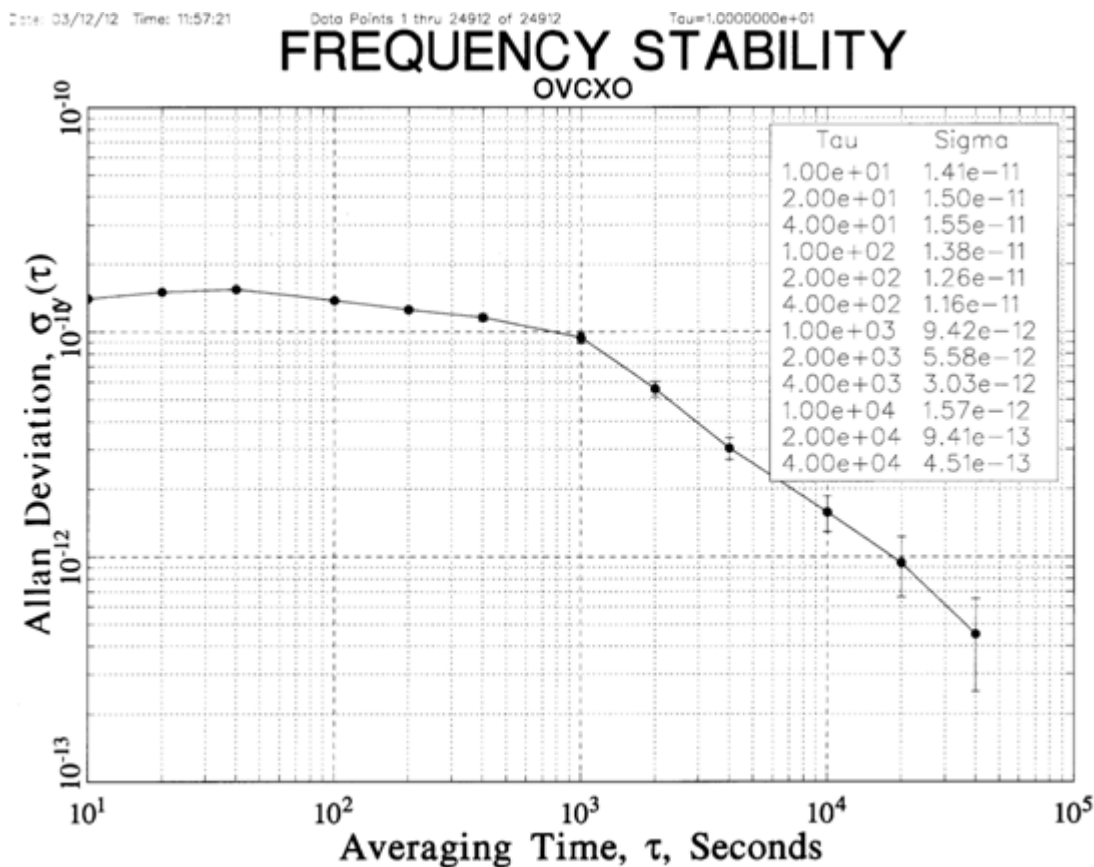


# Design Considerations For Optimizing Stability in GPS Disciplined Frequency Standards



## 1. Introduction

Advances in technology tend to be iterative, that is, as capability increases additional advances become possible that increase capability further.

This is particularly true in the communications market sectors, where telecommunications has advanced at a rapid pace that in many areas of the world 3G is not yet available, but we're already at a point where systems are rapidly migrating to 4G.

Equally in SatCom and Broadcasting markets, data rates have soared to a point where high definition TV is now available to a majority of households in the US, and around the world in developed countries.

Behind this progress, at system and component level it is important to provide the support to make these advances possible in terms of accuracy, stability, and phase noise of the critical signals driving the systems.

While the GPS system has been established for several decades, the equipment that utilizes the precise output data from the satellites has grown not only in massive numbers and variety, but also in its role to support key infrastructure elements and more demanding performance capability. In parallel with this, the GPS system has also developed in terms of specification and capability delivered from the satellites.

Specifically this paper describes one aspect of the detailed measures taken to insure the GPS satellite capability is converted to the best frequency and timing capability possible, to provide an economical frequency standard designed to meet the demanding requirements of today's technological advances.

## 2. System Overview.

In essence, the system architecture is relatively simple, consisting of just three main system building blocks, a GPS Receiver "Engine", a high performance oscillator, and some electronics to implement a phase lock loop in order to "discipline" (phase lock) the oscillator to the GPS engine output.

As usual, the devil is in the details!

While the GPS "Engine" and the high performance oscillator both warrant detailed examination and explanation in their own right, the subject of this paper is focused on designing of a phase lock loop that will extract the last ounce of performance from the satellite signals, and deliver the best possible performance to end user applications.

### 3. GPS Receiver Phase Lock Loop Design

This section describes the design and performance of a 1PPS phase-lock loop for a GPS frequency standard utilizing a high quality ovenized quartz crystal oscillator (OVCXO). Such oscillators are design to provide excellent phase noise (subject of another paper) and short term stability performance.

The loop compensation is basically a lag-lead filter with an added low-pass filter to remove the GPS receiver “noise”.

The desired transfer functions are first described in the frequency domain with Laplace operators and the bilateral transform is used to map the functions from the S plane into the Z plane.

The difference equation, which is ultimately used by the software implementing the control, is derived from :

$$H(z) = Y(z)/X(z)$$

where :

H = transfer function

Y = Output

X = Input

$z^{-1}$  represents a delay.

The loop is optimized using a model in a circuit simulator in both the frequency domain and the time domain. Higher order control loops have been considered, but do not add any significant performance.

## 4. Phase Lock Loop (PLL)

Although the oscillator generated frequency is nominally 10MHz, as the output used from the GPS engine is a one pulse per second (1PPS) signal, it is convenient to divide the 10MHz by  $10^7$  and lock the resultant 1Hz pulse to the GPS 1PPS.

The chosen loop compensation transfer function is:

$$H(s) = \frac{\tau_z s + 1}{\tau_p s}$$

where:

$s$  = Laplace operator

$\tau_z$  = time constant of zero

$\tau_p$  = time constant of pole

Mapping into the z plane we use the bi-lateral transform:

$$s = \frac{2(1 - z^{-1})}{T(1 + z^{-1})}$$

where:

T= sampling rate (1 second in all cases)

This results in:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{2\tau_z + T + (T - 2\tau_z)z^{-1}}{2\tau_p(1 - z^{-1})}$$

With  $z^{-1}$  equivalent to a delay in the time domain, the difference equation (used in the software portion of the loop) is:

$$Y_n = Y_{n-1} + k_1 X_n + k_2 X_{n-1}$$

where :

$$k_1 = \frac{2\tau_z + T}{2\tau_p} \quad \text{and} \quad k_2 = \frac{\left(\frac{T}{2} - \tau_z\right)}{\tau_p}$$

## 5. Low Pass Filter

One of the attributes of the 1PPS signal from the GPS receiver engine is fairly substantial phase variations, and therefore an additional one pole low-pass filter is necessary to attenuate these. The transfer function in the frequency domain is:

$$H(s) = \frac{1}{(\tau_l s + 1)}$$

again, using the bi-lateral transform we get:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{T(1 + z^{-1})}{(T - 2\tau_l)z^{-1} + T + 2\tau_l}$$

The resulting difference equation is:

$$Y_n = a_1 Y_{n-1} + a_2 (X_n + X_{n-1})$$

where:

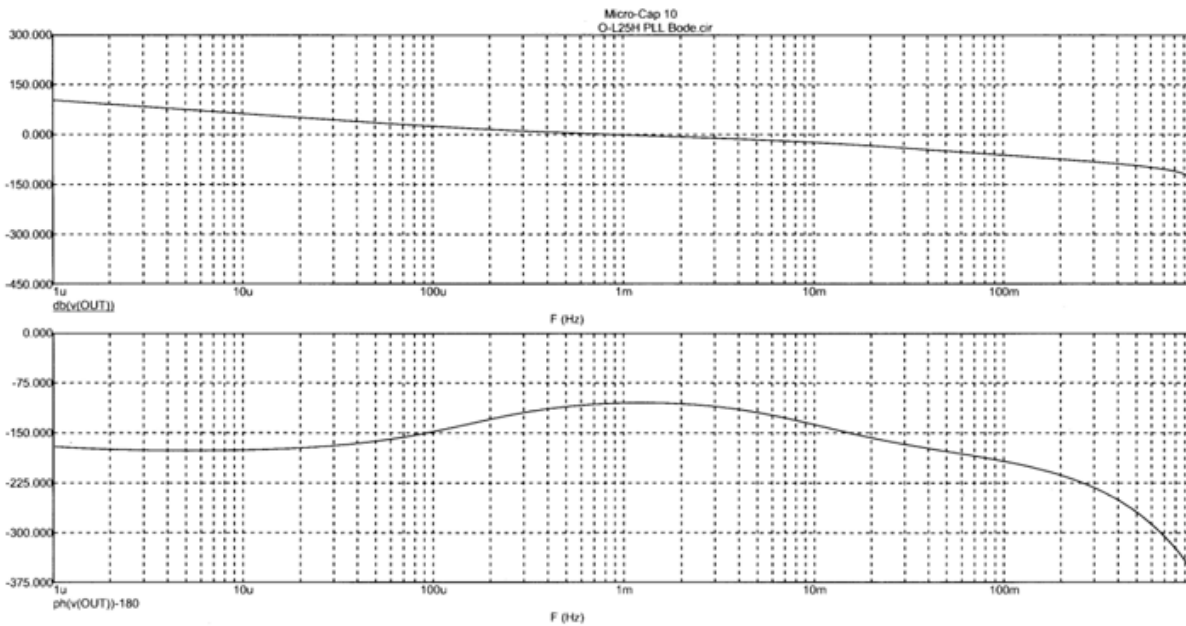
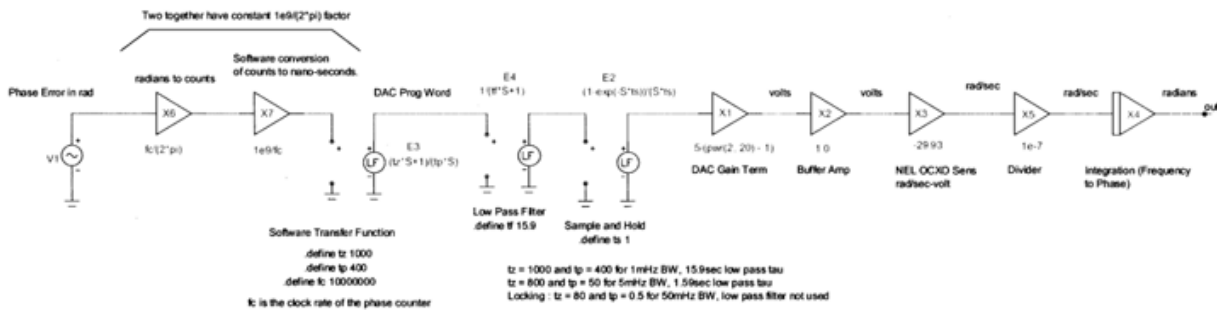
$$a_1 = \frac{(2\tau_l - T)}{(2\tau_l + T)} \quad \text{and} \quad a_2 = \frac{T}{(2\tau_l + T)}$$

Using an ovenized OCXO provides good stability out to approximately 100sec, therefore a slow loop with about 100 sec time constant is required.

The pole of the low-pass filter must remain at least a decade higher in frequency than the loop bandwidth to preserve stability. Using a model in the Laplace frequency domain it can be determined through experimentation that a 5 mHz to 1 mHz bandwidth is adequate with normal room temperature variations. Higher bandwidths will result in increased short term noise from the GPS engine. A table of the various bandwidths is shown below:

BW (mHz)	$\tau_z$ (sec)	$\tau_p$ (sec)	$\tau_l$ (sec)	$a_1$	$a_2$
50	80	0.5	--	--	--
5	800	50	1.59	0.521531	0.239234
1	1000	400	15.9	0.939024	0.030488

The MicroCap 10 model used for the PLL is illustrated below;



Open Loop Gain and Phase of 1PPS Control Loop with 1 mHz Bandwidth

In an instrument, implementation of the 1PPS loop with the OVCXO contains both the lag-lead filter and the low-pass filter as depicted above. In the real world, the instrument is clearly required to deliver an accurate, stable signal in the shortest possible time from power on, and therefore, in order to minimize the "time to lock" there are two modes of operation, acquisition and locked.

The acquisition mode has a wider bandwidth and no low-pass filtration, allowing the instrument to quickly "home in" to approximate values before switching to the "locked" mode.

The locked mode has a narrower bandwidth and includes the low-pass filter, providing finer resolution at a much slower pace, maximizing the delivered performance.

Once in "locked" mode, there are two separate loop bandwidth implementations, dependent upon whether or not the instrument is within a  $\pm 100$ nsec window. Outside of this window, the loop bandwidth is 5mHz, again allowing for much faster phase error settling, whereas within the 100nsec window the loop bandwidth applied is 1mHz, in order to deliver the ultimate in performance once the loop has settled.

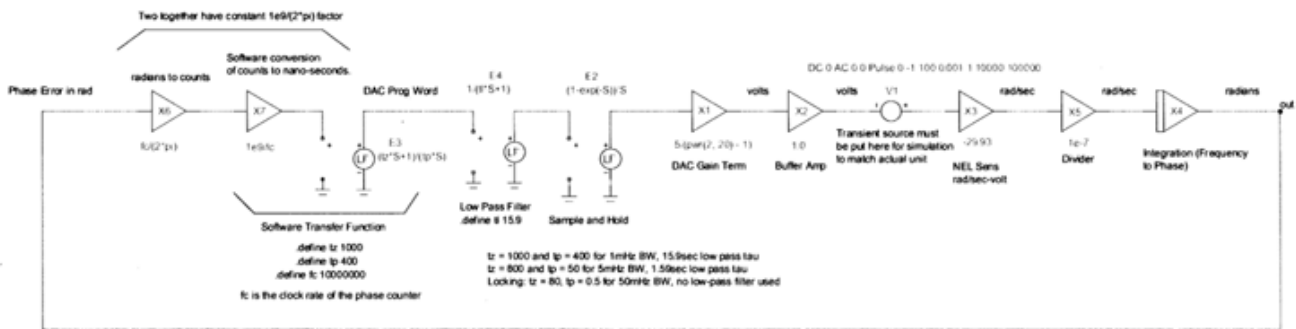
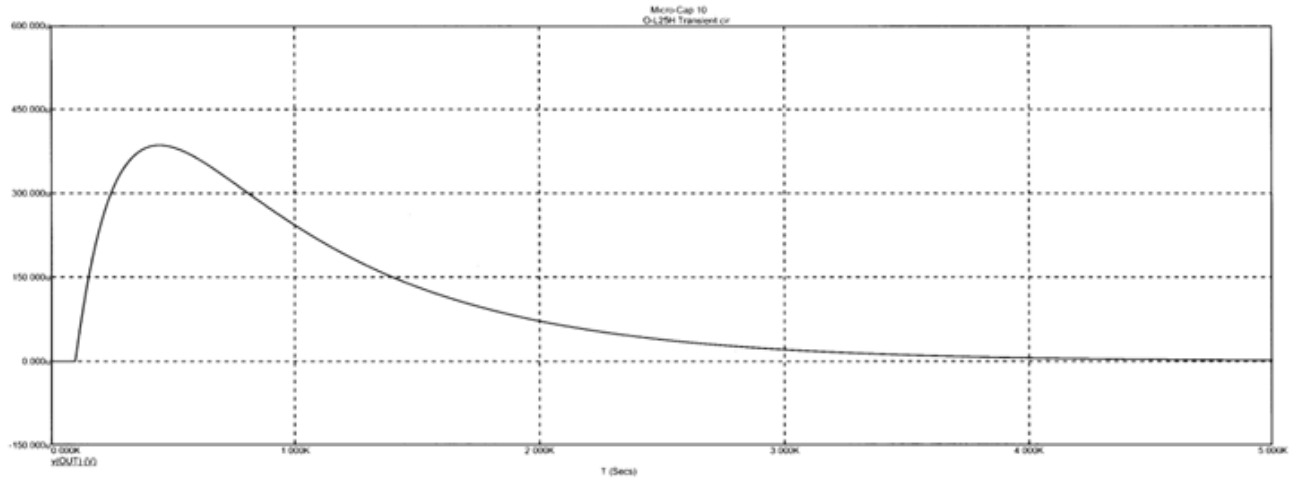
Following the low-pass filter is a sample and hold transfer function where T is the sample period equal to 1 sec:

$$H(s) = \frac{1 - e^{-sT}}{sT}$$

In practice it does not add any significant phase at the crossover frequency however is included for completeness.

To drive the OVCXO control voltage a DAC with at least 20 bit resolution, should be used. This equates to approximately 5 $\mu$ V per step, resulting in approximately  $2.4 \times 10^{-12}$   $\Delta f/f$  per step with a 30 rad/sec/volt OVCXO varactor sensitivity.

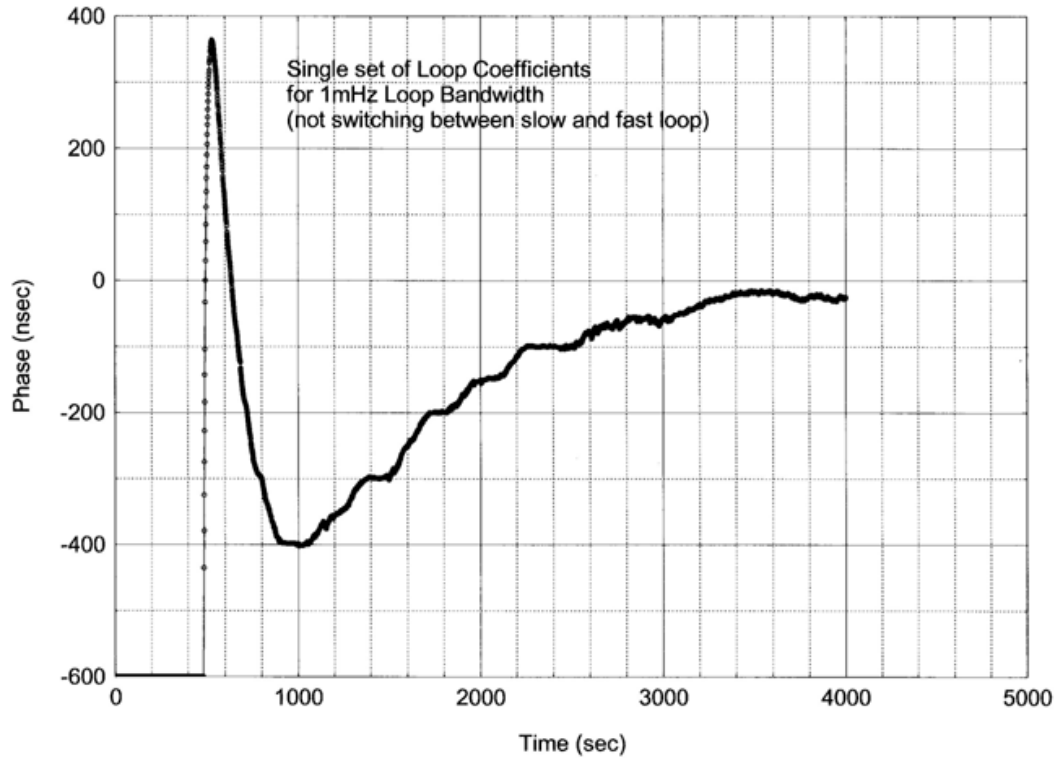
A simulated closed-loop transient response and measured transient response are shown below. Both plots show that it takes about 3500 sec to settle to zero phase error with a step in the varactor voltage as the excitation.



Simulated Transient Response of 1PPS Phase-Lock Loop with 1 mHz Bandwidth



## OVCXO Transient Response



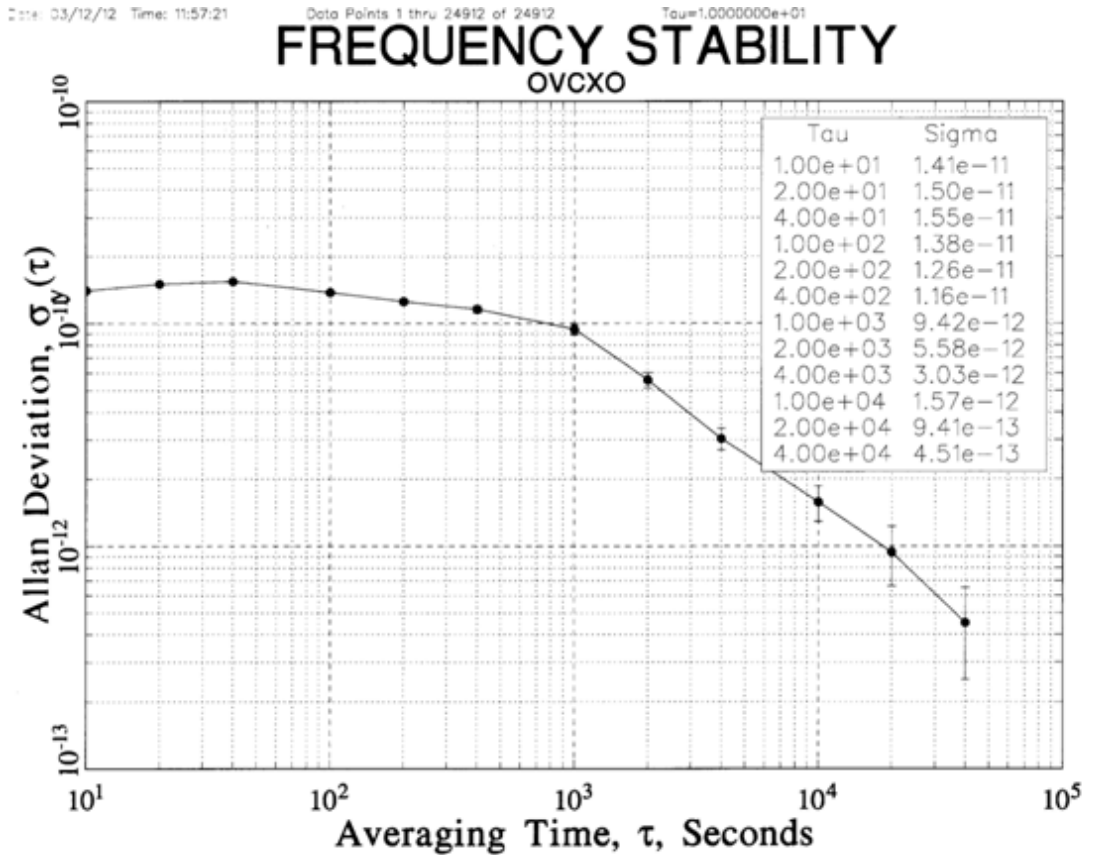
### Measured Transient response

The most widely used method for measuring the actual frequency stability of precision frequency standards is a statistical method called Allan Deviation (developed by David Allan).

Further information on this is available at :

[http://www.ptfinc.com/dsheets/ptf\\_Frequency%20and%20Time%20Handbook.pdf](http://www.ptfinc.com/dsheets/ptf_Frequency%20and%20Time%20Handbook.pdf)

A plot showing the resultant Allan Deviation is shown below:



### Short Term Stability with 1 mHz Loop Bandwidth

Above about 159 sec the stability of the GPS engine 1PPS takes over, whereas below 159 sec the OVCXO stability is seen, providing a much better performance realizable in the short term.

This is confirmed by the following equation showing the loop response time for a 1mHz bandwidth:

$$\tau = \frac{1}{2\pi f_{BW}} = \frac{1}{2\pi (1mHz)} = 159 \text{ sec}$$

It is therefore necessary to have an OVCXO with good short term stability in order to achieve the stability performance shown above.

For further information please go to:

<http://www.ptfinc.com/>